PG – 145

First Semester M.Sc. Examination, January 2016
(CBCS)
MATHEMATICS
M104T : Ordinary Differential Equations

Time : 3 Hours

Max. Marks : 70

Instructions : 1) *Answer **any five** questions.*
2) ***All** questions carry **equal** marks.*

1. a) Establish Abel's formula for $L_n y = 0$ defined on I and discuss one of its consequences. 8

   b) Find the Wronskian of independent solutions of a homogeneous differential equation defined on [–1, 1] which has a characteristic root M_1 of multiplicity three. 6

2. a) Define adjoint and self-adjoint differential equations. Prove that $L_n^{**} = L_n$. 6

   b) Solve by the method of variation of parameters $y'' + y = \frac{1}{1+\sin x}$. 4

   c) Show that the solution $\phi(x)$ of $\{p(x)y'\}' + q(x)y = 0$ defined on I has only a finite number of zeros in any bounded closed sub-interval of I. 4

3. a) State the existence and uniqueness theorem on the solution of a first order IVP and illustrate it for $y' = y^2$; $y(1) = -1$ in the domain $|x-1| \le a, |y+1| \le b$. 8

   b) For the IVP $y' = 2\sqrt{y}$; $y(0) = 0$, show that the uniqueness of the solution in the Picard's theorem fails when the Lipschitz condition is dropped. 6

4. a) Define Sturm-Liouville eigen value problem and show that the eigen values of this problem are simple. 6

   b) Solve : $y'' + \lambda y = x$; $y(0) = 0 = y(1)$ by constructing its Green's function. 8

5. a) Find the ordinary, regular and irregular singular points (including ∞) of $xy'' + (1-x)y' + ny = 0$ (n is a constant). 6

   b) Obtain the polynomial solution of Laguerre's differential equation about x = 0 and further show that $L_n(x) = \frac{e^x}{\underline{|n}} \frac{d^n}{dx^n}(x^n e^{-x})$. 8

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6. a) Reduce Chebyshev's differential equation into one with constant co-efficients and hence obtain its general solution. 5

   b) Obtain the general solution of Gauss' hyper geometric equation about $x = 0$ and hence obtain its solution about $x = 1$ using appropriate transformation. 9

7. a) Find the fundamental matrix and also determine exp (At) of the matrix equation.

$$\frac{dx}{dt} = A\underset{-}{x}, \text{ where } A = \begin{bmatrix} -1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & 3 & 0 \end{bmatrix}.$$ 8

   b) Obtain the solution $\psi(t)$ of the IVP

$$\frac{dx}{dt} = A\underset{-}{x} + B(t) \; ; \; \psi(0) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

where $A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}$ and $B(t) = \begin{bmatrix} \sin t \\ \cos t \end{bmatrix}$. 6

8. a) Determine the stability and nature of the critical point (0, 0) of the system.

$$\frac{dx}{dt} = x + 2y + x\cos y \; ; \; \frac{dy}{dt} = -y - \sin y.$$ 6

   b) Construct Liapunov function and hence determine the stability of the critical point (0, 0) of the non linear systems.

i) $\frac{dx}{dt} = -x + 2x^2 + y^2 \; ; \; \frac{dy}{dt} = -y + xy.$

ii) $\frac{dx}{dt} = x^3 - 3xy^4 \; ; \; \frac{dy}{dt} = x^2y - 2y^3 - y^5.$ 8